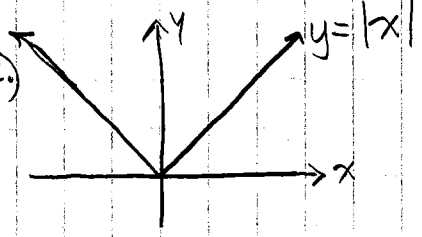
Pre-Calculus 11 8.1

1. a) $y = |x|$ b.) positive c.)



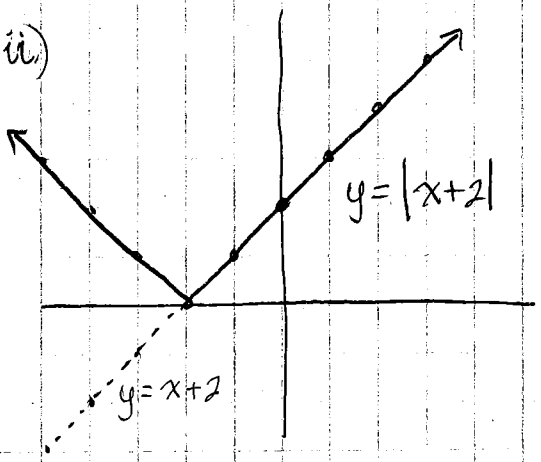
2. a) i) x-intercept → $y = 0$

$y = |x+2|$
$0 = |x+2|$
$0 = x+2$
$x = -2$

y-intercept → $x = 0$

$y = |x+2|$
$y = |0+2|$
$y = |2|$
$y = 2$

ii)



iii) D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} x+2 \text{ if } x \geq -2 \\ -(x+2) \text{ if } x < -2 \end{cases}$

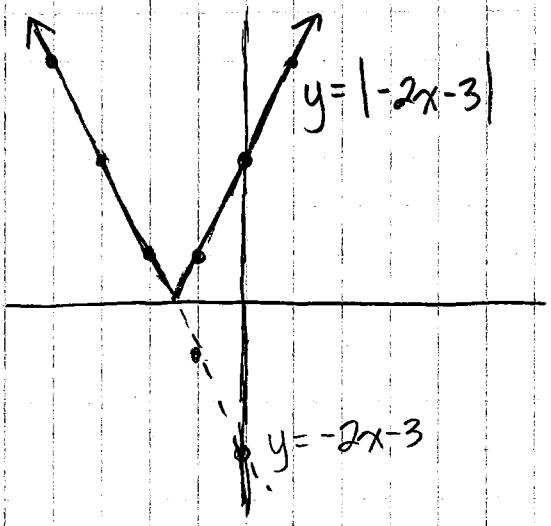
b.) i) x-int → $y = 0$

$y = |-2x-3|$
$0 = |-2x-3|$
$0 = -2x-3$
$3 = -2x$
$x = \frac{3}{-2} = -1.5$

y-int → $x = 0$

$y = |-2x-3|$
$y = |-2(0)-3|$
$y = |-3|$
$y = 3$

ii)



iii) D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} -2x-3 \text{ if } x \leq -1.5 \\ -(-2x-3) = 2x+3 \text{ if } x > -1.5 \end{cases}$

PC11 8.1 con't... 2

2. c.) i.) x-int → y=0

$y = |2x+3|$

$0 = |2x+3|$

$0 = 2x+3$

$-3 = 2x$

$x = -\frac{3}{2} = -1.5$

y-int → x=0

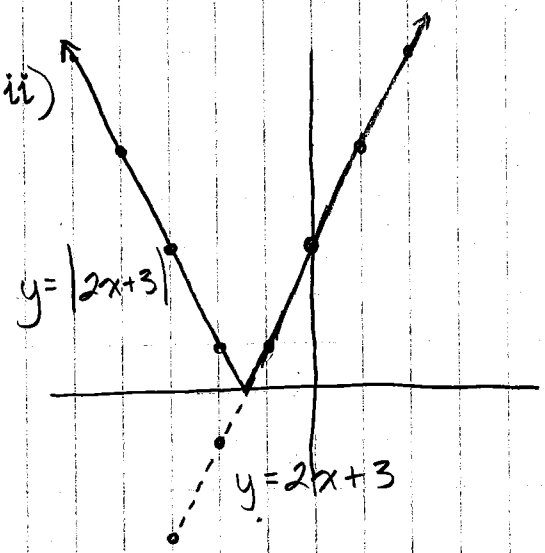
$y = |2x+3|$

$y = |2(0)+3|$

$y = |3|$

$y = 3$

ii)



iii) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} 2x+3 & \text{if } x \geq -1.5 \\ -(2x+3) & \text{if } x < -1.5 \end{cases}$

d.) i.) x-int → y=0

$y = |3-x|$

$0 = |3-x|$

$0 = 3-x$

$x = 3$

y-int → x=0

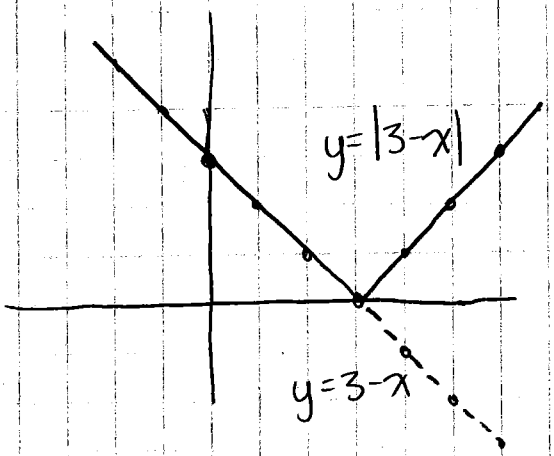
$y = |3-x|$

$y = |3-0|$

$y = |3|$

$y = 3$

ii.)



iii.) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} 3-x & \text{if } x \leq 3 \\ -(3-x) = x-3 & \text{if } x > 3 \end{cases}$

e.) i.) x-int → y=0

$y = |2+\frac{1}{2}x|$

$0 = |2+\frac{1}{2}x|$

$0 = 2+\frac{1}{2}x$

$-2 = \frac{1}{2}x$

$2 \cdot -2 = x$

$-4 = x$

y-int → x=0

$y = |2+\frac{1}{2}x|$

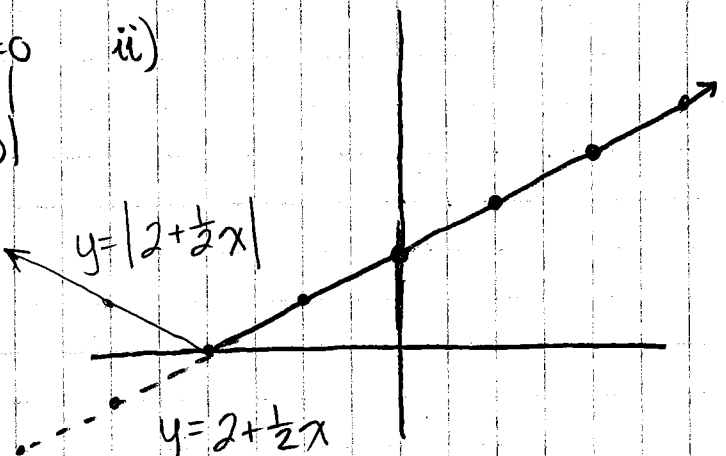
$y = |2+\frac{1}{2}(0)|$

$y = |2+0|$

$y = |2|$

$y = 2$

ii)



iii.) D = $\{x \mid x \in \mathbb{R}\}$

R = $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv) $y = \begin{cases} 2+\frac{1}{2}x & \text{if } x \geq -4 \\ -(2+\frac{1}{2}x) & \text{if } x < -4 \end{cases}$

PC 11 §1.1 con't... 3

2. f.) i.) x-int $\Rightarrow y=0$

$y=\left|-\frac{2}{3}x+3\right|$

$0=\left|-\frac{2}{3}x+3\right|$

$0=-\frac{2}{3}x+3$

$\frac{2}{3}x=3$

$x=3\cdot\frac{3}{2}$

$x=\frac{9}{2}=4.5$

y-int $\rightarrow x=0$

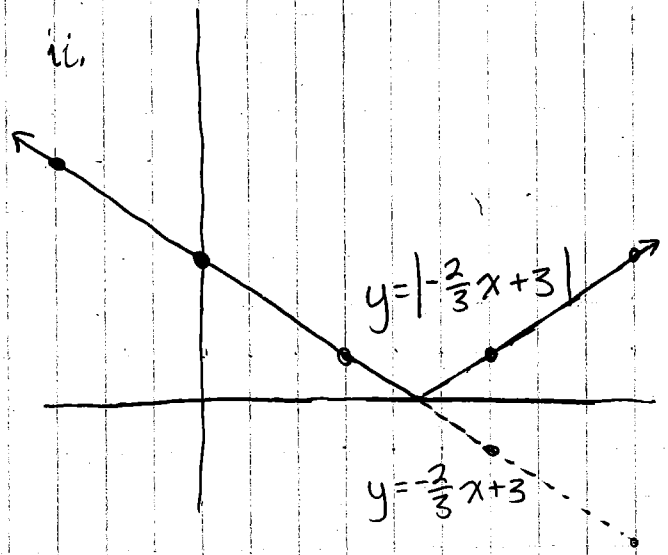
$y=\left|-\frac{2}{3}x+3\right|$

$y=\left|-\frac{2}{3}(0)+3\right|$

$y=|3|$

$y=3$

ii.

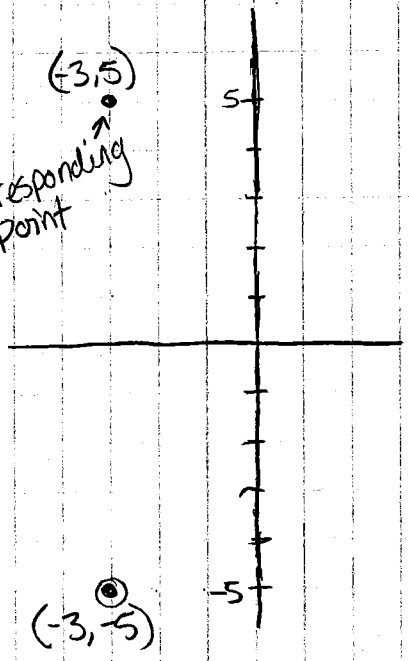


iii) D: $\{x \mid x\in\mathbb{R}\}$

R: $\{y \mid y\geq 0,\ y\in\mathbb{R}\}$

iv) $y=\begin{cases} -\frac{2}{3}x+3 & \text{if } x\leq 4.5 \\ -\left(-\frac{2}{3}x+3\right)=\frac{2}{3}x-3 & \text{if } x>4.5 \end{cases}$

3.



$y=f(x)$

$-5=f(-3)$

$\downarrow$

$y=|f(x)|$

$5=|f(-3)|$

4. $y=f(x) \rightarrow$ x-int $=-2$ and y-int $=-5$

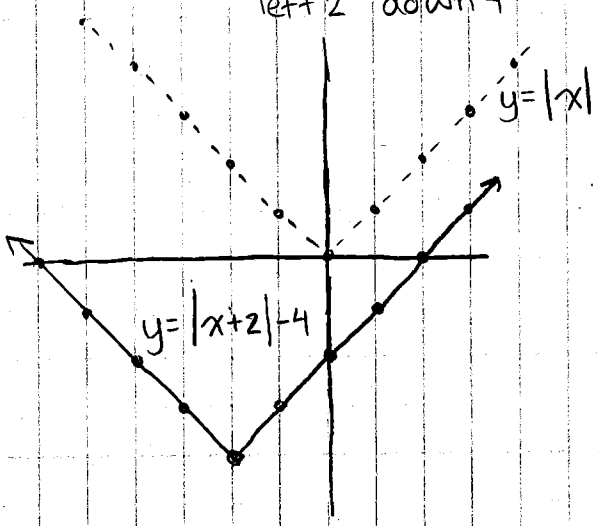
$y=|f(x)| \rightarrow$ x-int $=-2$ (critical point)

y-int $=|-5|=5$

PC11 8.1 con't... 4

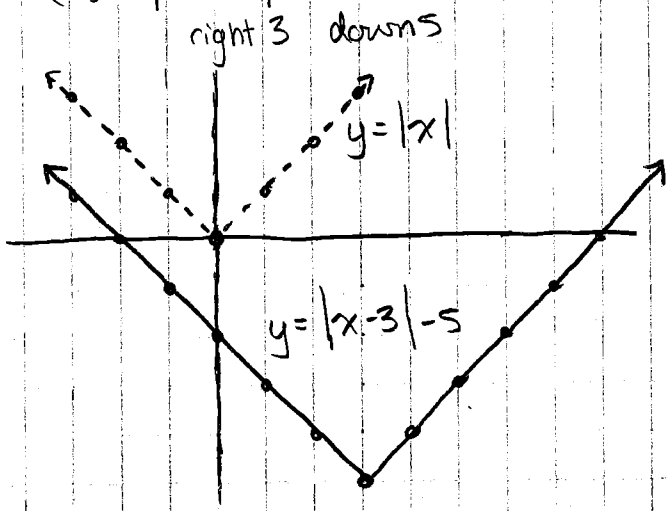
5. a.) $f(x) = |x+2| - 4$

left 2, down 4



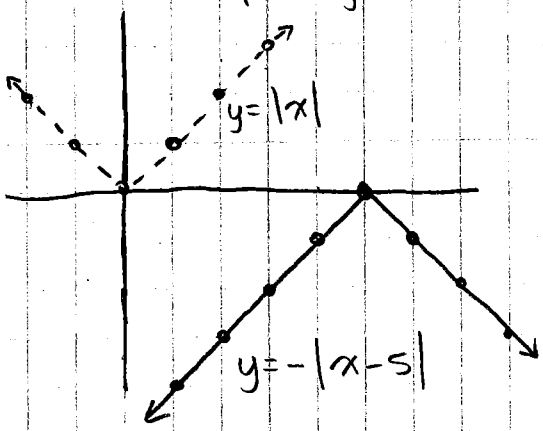
b.) $f(x) = |x-3| - 5$

right 3, down 5



c.) $f(x) = -|x-5|$

flip, right 5



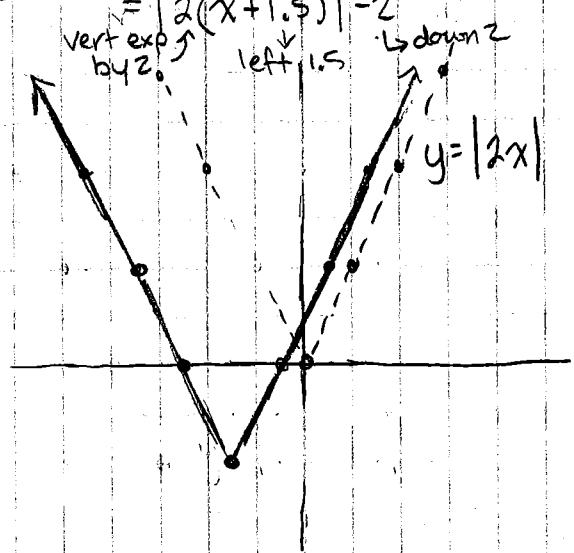
d.) $y = 3 - |x+4| = -|x+4| + 3$

flip, left 4, up 3

e) $f(x) = |2x+3| - 2$

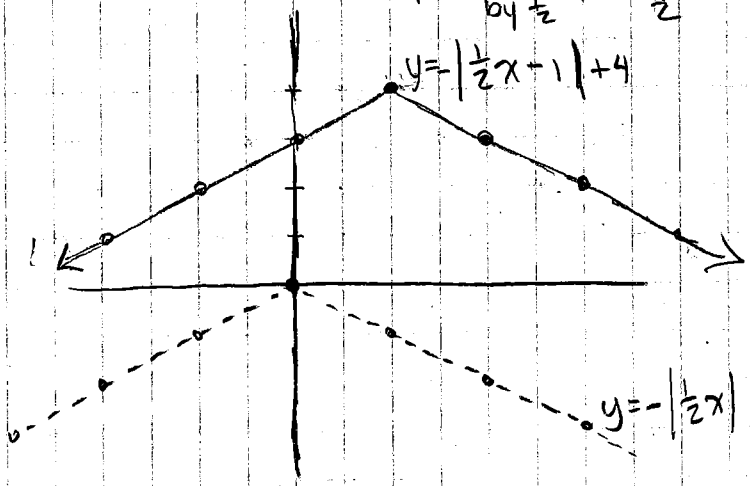
$= |2(x+1.5)| - 2$

vert exp by 2, left 1.5, down 2



f.) $f(x) = -\left|\frac{x}{2} - 1\right| + 4 = -\left|\frac{1}{2}(x-2)\right| + 4$

flip, vert comp by $\frac{1}{2}$, right 2, up 4



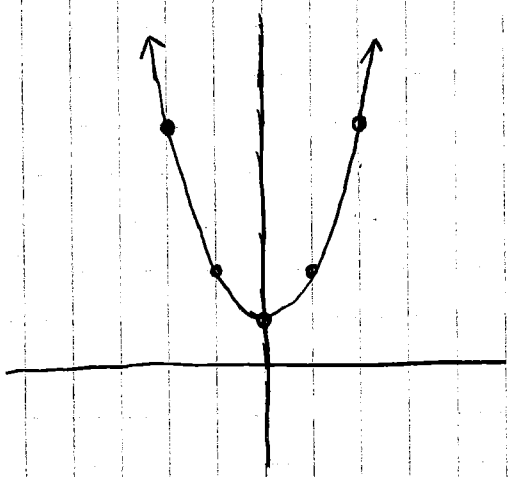
Pre-Calculus 11 8.1 part 2

1. a) $y = |x^2 + 1|$

i.) x-int → $y = 0$
$0 = |x^2 + 1|$
$0 = x^2 + 1$
$-1 = x^2$
$\sqrt{-1} = x$
∴ no x-ints

y-int → $x = 0$
$y = |0^2 + 1|$
$y = 1$

ii.)



iii.) $D: \{x \mid x \in \mathbb{R}\}$
$R: \{y \mid y \geq 1, y \in \mathbb{R}\}$

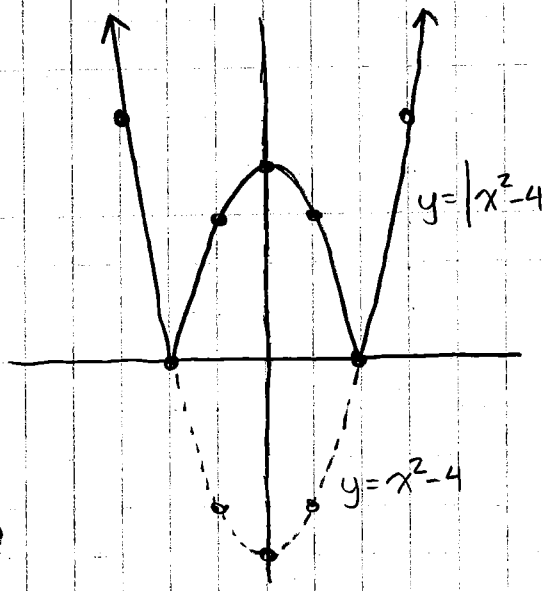
iv.) $y = x^2 + 1$ for $x \in \mathbb{R}$

b.) $y = |x^2 - 4|$

i.) x-int → $y = 0$
$0 = |x^2 - 4|$
$0 = x^2 - 4$
$4 = x^2$
$\pm\sqrt{4} = x$
$x = \pm 2$

y-int → $x = 0$
$y = |0^2 - 4|$
$y = |-4|$
$y = 4$

ii.)



iii) $D: \{x \mid x \in \mathbb{R}\}$
$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

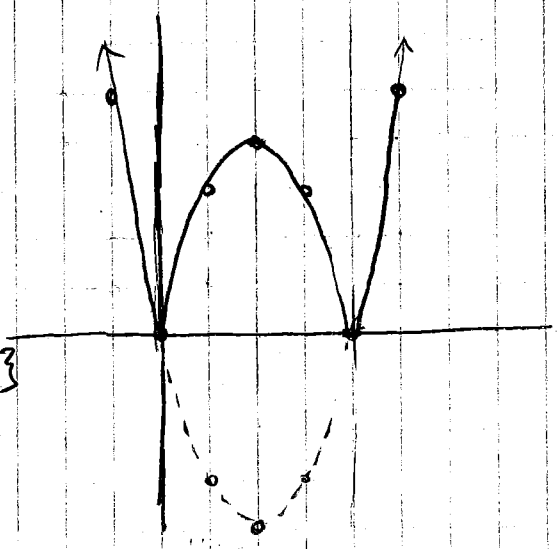
iv) $y = \begin{cases} x^2 - 4 & \text{if } x \leq -2 \text{ and } x \geq 2 \\ -(x^2 - 4) & \text{if } -2 < x < 2 \end{cases}$

c) $y = |(x-2)^2 - 4|$

i.) x-int → $y = 0$
$0 = (x-2)^2 - 4$
$4 = (x-2)^2$
$\pm\sqrt{4} = x - 2$
$\pm 2 + 2 = x$
$x = 2 + 2 = 4$
$x = 2 - 2 = 0$

y-int → $x = 0$
$y = |(0-2)^2 - 4|$
$y = |(-2)^2 - 4|$
$y = |4 - 4| = 0$

ii.)



iii.) $D: \{x \mid x \in \mathbb{R}\}$
$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} (x-2)^2 - 4 & \text{if } x \leq 0 \text{ and } x \geq 4 \\ -(x-2)^2 + 4 & \text{if } 0 < x < 4 \end{cases}$

PC11 8.1 part 2 cont... 2

1. d.) $y = \left|\frac{1}{2}(x-3)^2 - 8\right|$

i.) x-int → $y=0$

$0 = \frac{1}{2}(x-3)^2 - 8$

$8 = \frac{1}{2}(x-3)^2$

$16 = (x-3)^2$

$\pm\sqrt{16} = x-3$

$3 \pm 4 = x$

$x = 3+4 = 7$

$x = 3-4 = -1$

y-int → $x=0$

$y = \left|\frac{1}{2}(0-3)^2 - 8\right|$

$y = \left|\frac{1}{2}(-3)^2 - 8\right|$

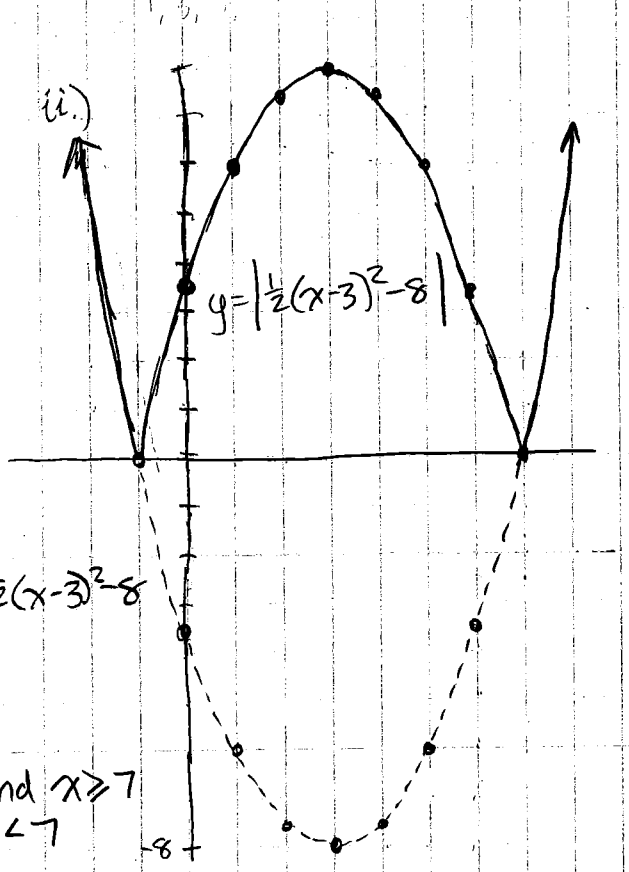
$y = \left|\frac{1}{2}(9) - 8\right|$

$y = |4.5 - 8|$

$y = |-4.5|$

$y = 4.5$

ii.)



iii.) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} \frac{1}{2}(x-3)^2 - 8 & \text{if } x \leq -1 \text{ and } x \geq 7 \\ -\frac{1}{2}(x-3)^2 + 8 & \text{if } -1 < x < 7 \end{cases}$

e.) $y = |-2x^2 + 2|$

i.) x-int → $y=0$

$0 = -2x^2 + 2$

$\frac{-2}{-2} = \frac{-2x^2}{-2}$

$1 = x^2$

$\pm\sqrt{1} = x$

$x = 1, -1$

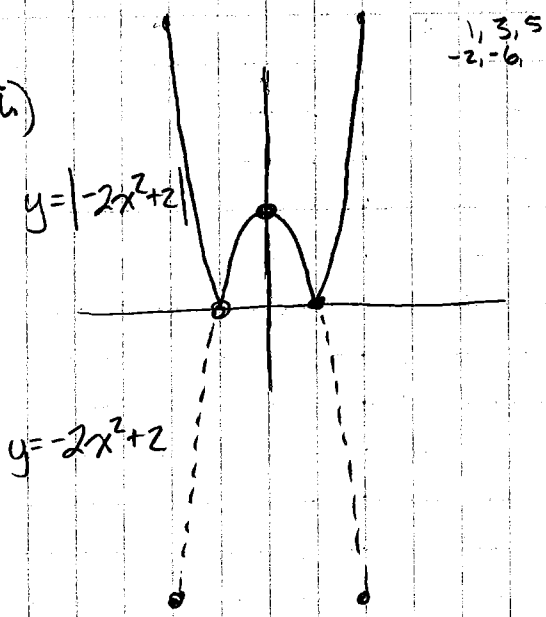
y-int → $x=0$

$y = |-2(0)^2 + 2|$

$y = |2|$

$y = 2$

ii.)



iii.) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

iv.) $y = \begin{cases} -2x^2 + 2 & \text{if } -1 \leq x \leq 1 \\ -(-2x^2+2) = 2x^2 - 2 & \text{if } x < -1 \text{ and } x > 1 \end{cases}$

PC11 8.1 part 2 con't...3

1, 3, 5
3, 9

1.f) $y = |-3(x+1)^2+3|$

i.) x-int → $y=0$

$0 = -3(x+1)^2+3$

$\frac{-3}{-3} = \frac{-3(x+1)^2}{-3}$

$1 = (x+1)^2$

$\pm\sqrt{1} = x+1$ → $x = -1+1 = 0$

$\pm 1 = x+1$ → $x = -1-1 = -2$

y-int → $x=0$

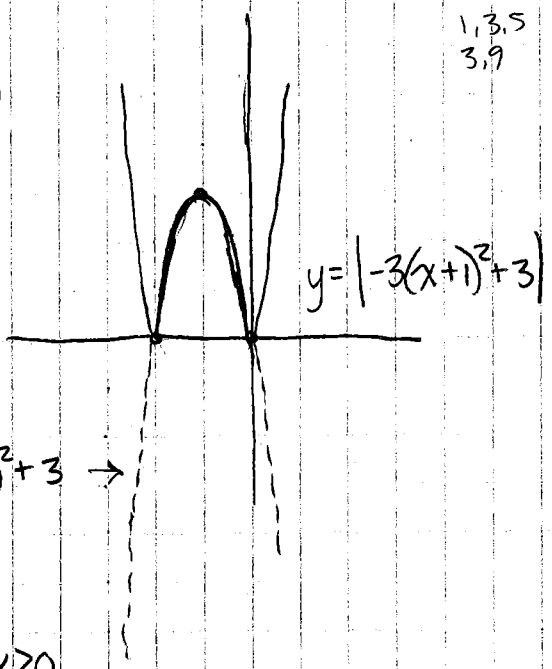
$y = |-3(0+1)^2+3|$

$y = |-3(1)+3|$

$y = |-3+3|$

$y = 0$

ii.)



iii) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

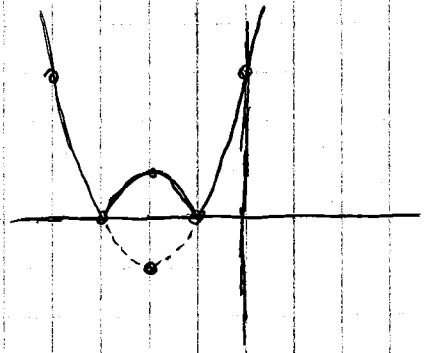
iv) $y = \begin{cases} -3(x+1)^2+3 & \text{if } -2 \leq x \leq 0 \\ 3(x+1)^2-3 & \text{if } x < -2 \text{ and } x > 0 \end{cases}$

2. a) $y = |x^2+4x+3|$ $\quad \frac{1}{2}(4)=2 \rightarrow 2^2=4$

$y = |(x^2+4x+4-4)+3|$

$y = |(x^2+4x+4)-4+3|$

$y = |(x+2)^2-1|$

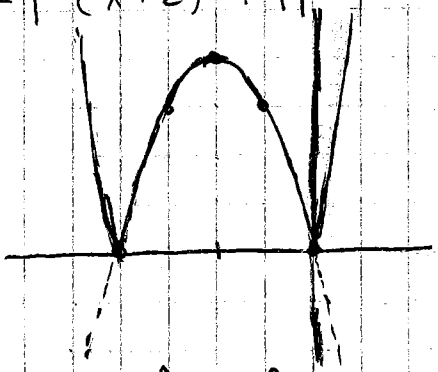


b) $y = |-x^2+4x|$ $\quad \frac{1}{2}(4)=2 \rightarrow 2^2=4$

$y = |-(x^2+4x+4-4)|$

$y = |-(x^2+4x+4)+4|$

$y = |-(x+2)^2+4|$



c) $y = |-3x^2-12x-15|$

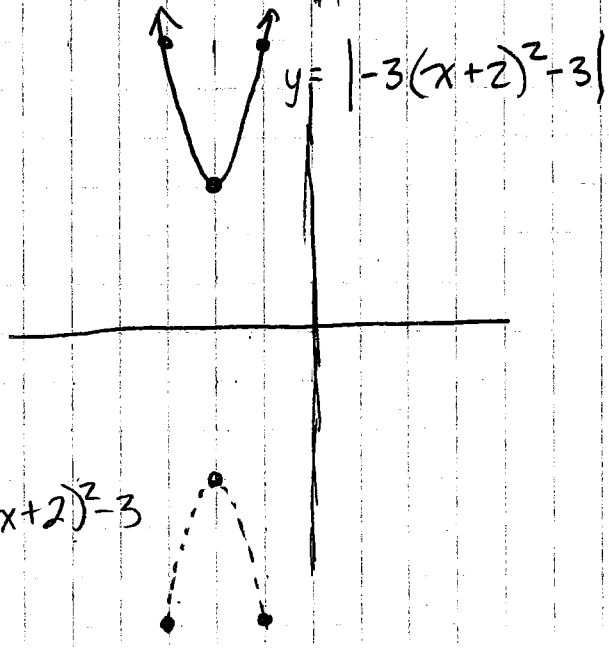
$y = |-3(x^2+4x)-15|$

$y = |-3(x^2+4x+4-4)-15|$

$y = |-3(x^2+4x+4)+12-15|$

$y = |-3(x+2)^2-3|$

$\leftarrow 2 \quad \downarrow 3$



PC11 8.1 pt 2 con't... 4

2.d.) $y=|-3x^2-12x-9|$ $\frac{1}{2}(4)=2 \rightarrow 2^2=4$

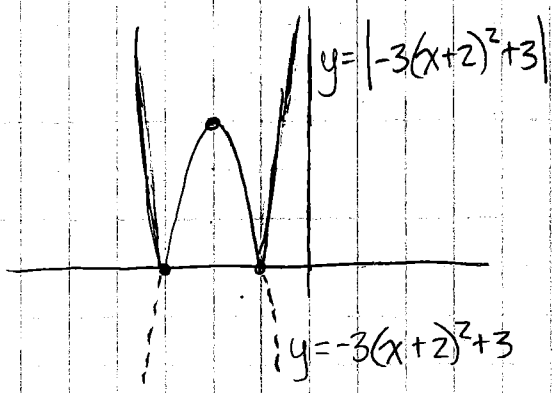
$y=|-3(x^2+4x)-9|$

$y=|-3(x^2+4x+4-4)-9|$

$y=|-3(x^2+4x+4)+12-9|$

$y=|-3(x+2)^2+3|$

flip ×3 ← 2 ↑3



e.) $y=\left|\frac{1}{2}x^2+2x-6\right|$ $\frac{1}{2}(4)=2 \rightarrow 2^2=4$

$y=\left|\frac{1}{2}(x^2+4x)-6\right|$

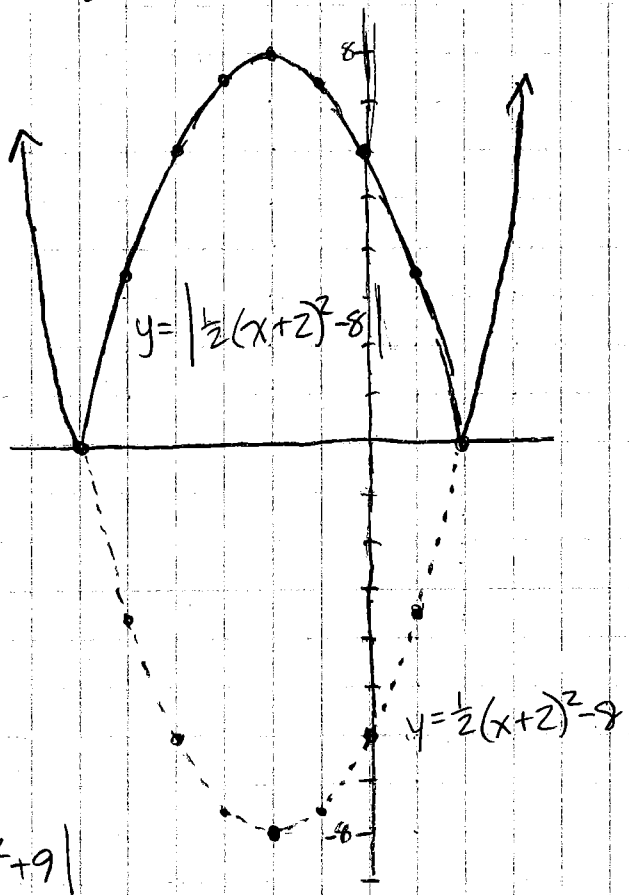
$y=\left|\frac{1}{2}(x^2+4x+4-4)-6\right|$

$y=\left|\frac{1}{2}(x^2+4x+4)-2-6\right|$

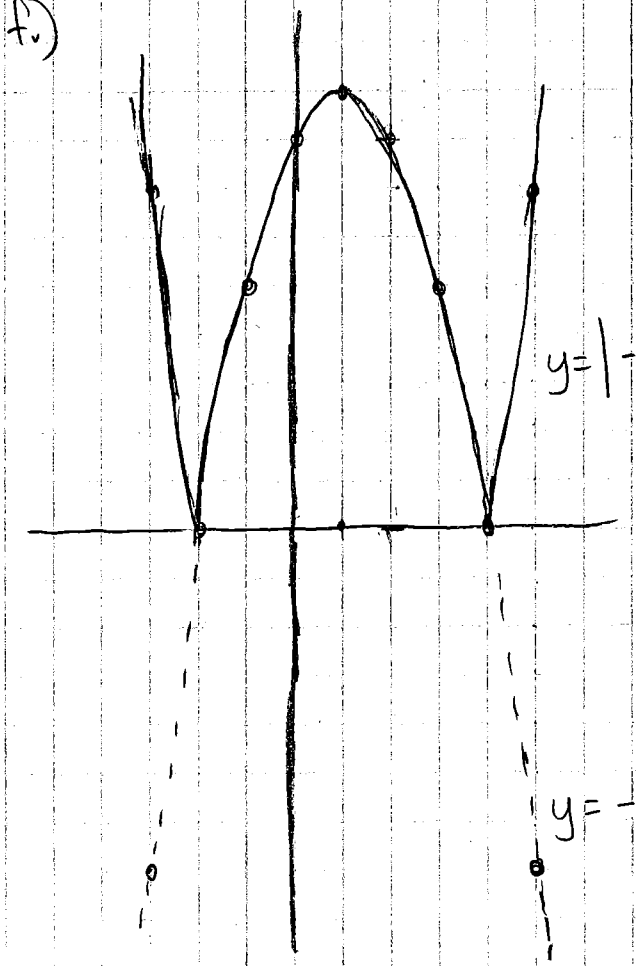
$y=\left|\frac{1}{2}(x+2)^2-8\right|$

×½ ← 2 ↓8

1,3,5,7,9



f.)



f.) $y=|-x^2+2x+8|$

$y=|-(x^2-2x)+8|$

$y=|-(x^2-2x+1-1)+8|$

$y=|-(x^2-2x+1)+1+8|$

$y=|-(x-1)^2+9|$

PC11 8.1 Pt2 cont...5

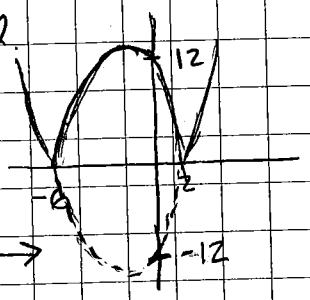
3. $f(x) = |x^2 + bx + c|$

positive - so original function $y = x^2 + bx + c$ opens up.

with a y-intercept $(0, -12)$ (x, y)

x-ints: -6 & 2

y-int: 12



Factored form of the quadratic $y = a(x - x_1)(x - x_2)$

x-intercepts: $x_1 = -6$, $x_2 = 2$

$y = a(x - (-6))(x - 2)$

$y = a(x + 6)(x - 2)$

Substitute point $(0, -12)$ (x, y)

$-12 = a(0 + 6)(0 - 2)$

$-12 = a(6)(-2)$

$\frac{-12}{-12} = \frac{a(-12)}{-12}$

$1 = a$

$y = 1(x + 6)(x - 2)$

$y = x^2 - 2x + 6x - 12$

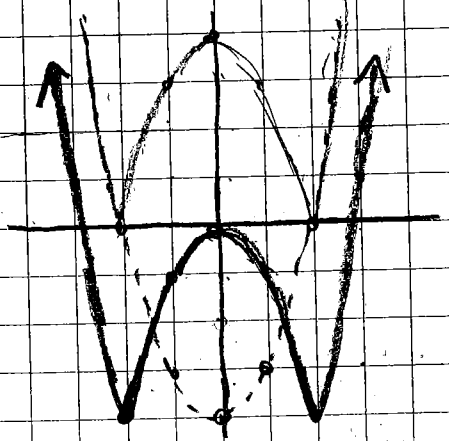
$y = x^2 + 4x - 12$

$\therefore f(x) = |x^2 + 4x - 12|$ so $b = 4$ & $c = -12$.

4.a) $f(x) = |x^2 - 4| - 4$

Graph $y = x^2$

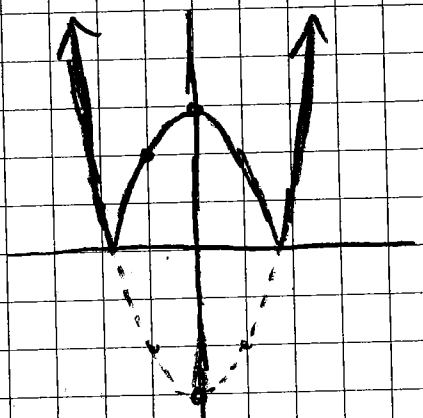
- move 4 down
- then abs value
- then 4 down



b) $f(x) = |x^2 - 3|$

Graph $y = x^2$

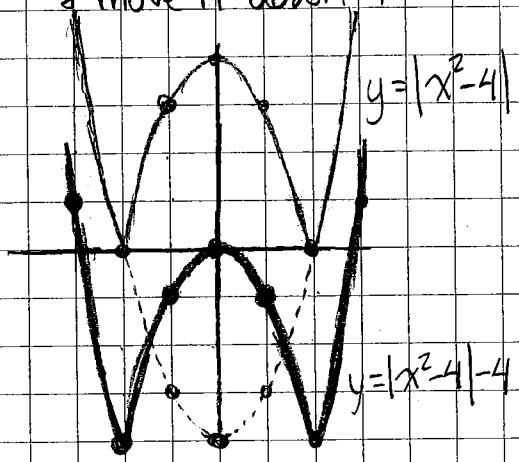
- move 3 down
- then abs value



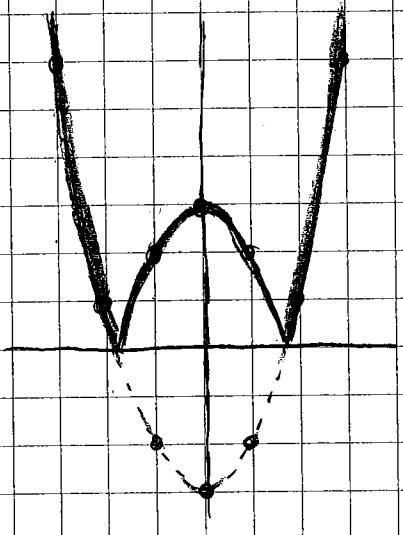
PC11 8.1 Pt2. con't...6

4.a) $f(x) = |x^2 - 4| - 4$

Graph $y = |x^2 - 4|$
& move it down 4



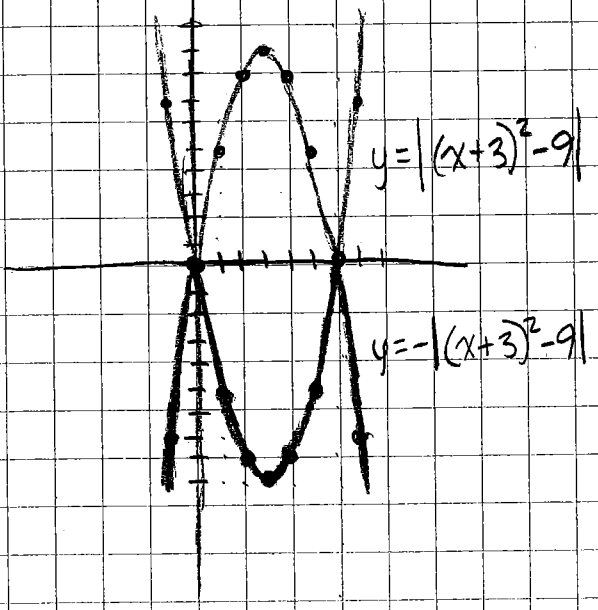
b.) $f(x) = |x^2 - 3|$



c.) $f(x) = -|x^2 + 6x|$

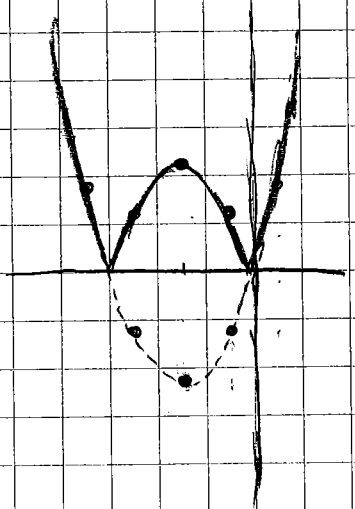
$\frac{1}{2}(6) = 3$
$\hookrightarrow 3^2 = 9$

Graph $y = |x^2 + 6x|$
$y = |x^2 + 6x + 9 - 9|$
$y = |(x+3)^2 - 9|$
Reflect it in the x-axis
for $y = -|(x+3)^2 - 9|$



d.) $f(x) = |x(x+3)|$
$= |x^2 + 3x|$
$= |x^2 + 3x + \frac{9}{4} - \frac{9}{4})$
$= |(x + \frac{3}{2})^2 - \frac{9}{4})$
$= |(x + 1.5)^2 - 2.25|$

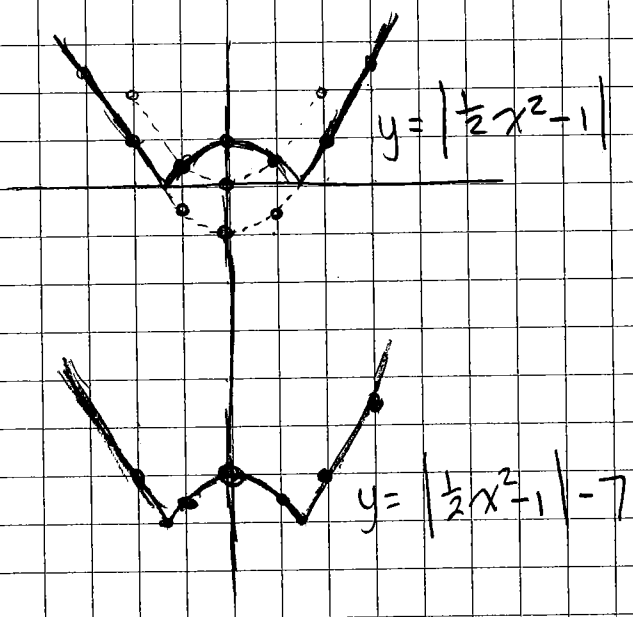
$\frac{1}{2}(3) = \frac{3}{2}$
$\hookrightarrow (\frac{3}{2})^2 = \frac{9}{4}$



PC 11 8.1 Pt 2 con't... 7

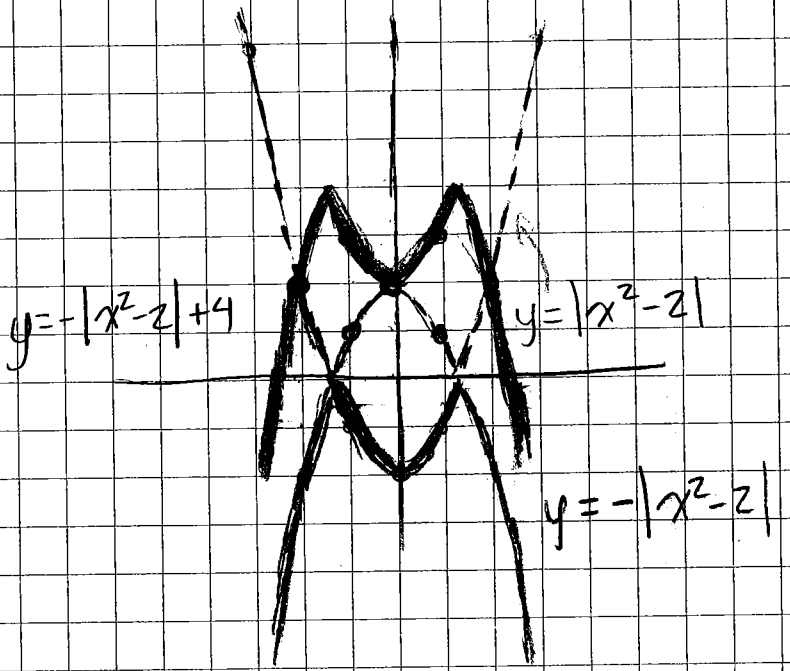
4e) $f(x) = \left|\frac{1}{2}x^2 - 1\right| - 7$

Steps: ① Graph $y = \frac{1}{2}x^2$
② Move it down 1
③ Take absolute value
④ Move down 7



4f) $f(x) = -|x^2 - 2| + 4$

Steps ① Graph $y = x^2 - 2$
② Take absolute value
③ Reflect in x-axis
(ie "flip it")
④ Move up 4.

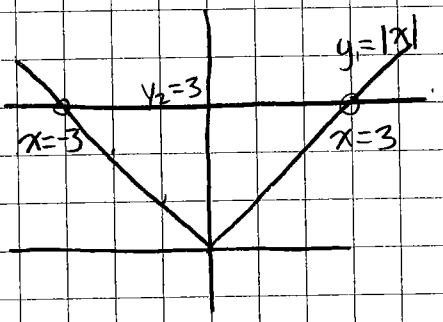


Pre-Calculus 11 8.2

1. a) $|x| = 3$

$y_1 = |x|$

$y_2 = 3$



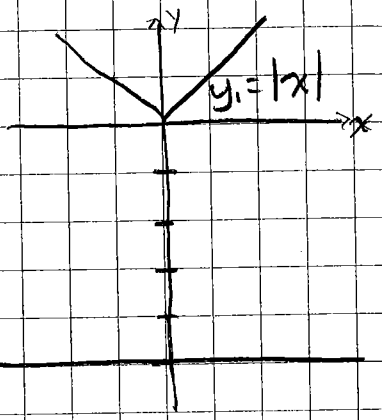
b) $|x| + 5 = 0$

$|x| = -5$

$y_1 = |x|$

$y_2 = -5$

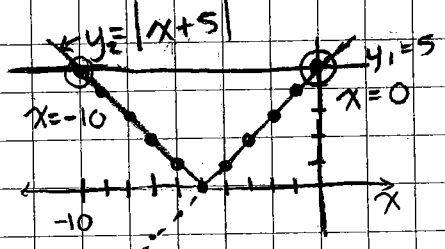
∴ No Solution



c) $5 = |x+5|$

$y_1 = 5$

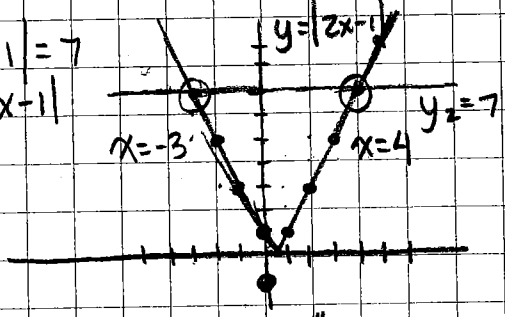
$y_2 = |x+5|$



d) $|2x-1| = 7$

$y_1 = |2x-1|$

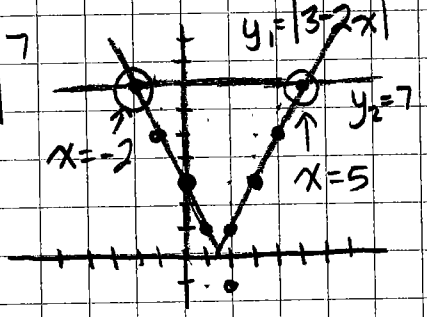
$y_2 = 7$



e) $|3-2x| = 7$

$y_1 = |3-2x|$

$y_2 = 7$



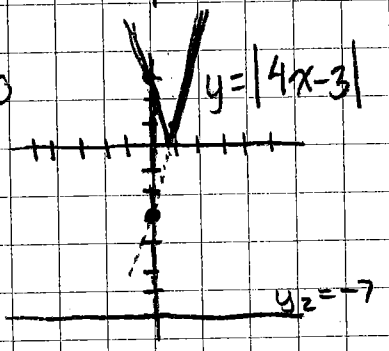
f) $|4x-3| + 7 = 0$

$|4x-3| = -7$

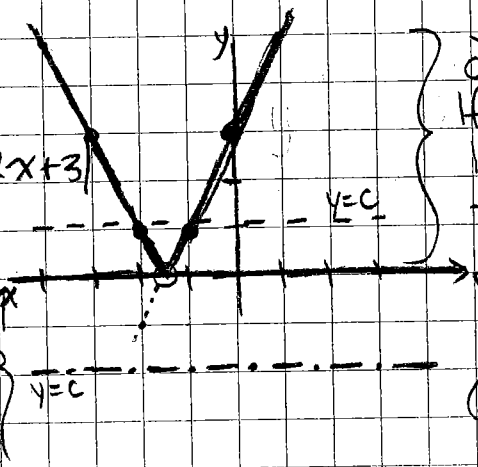
$y_1 = 4x-3$

$y_2 = -7$

∴ No Solution.



2. $|2x+3| = c$



a) $c > 0$

If $c > 0$, then the horizontal line $y = c$ will intersect twice creating two solutions

b) $c = 0$

If $c = 0$, then the horizontal line $y = c$ will intersect once (at the vertex) creating one solution

c) $c < 0$

If $c < 0$, the horizontal line $y = c$ will not intersect ever, so no solutions

PC11 8.2 con't...2

3.a) $\left|\frac{x}{3}-\frac{1}{2}\right|=1$

Case #1

$\frac{x}{3}-\frac{1}{2}=1$

$6\left(\frac{x}{3}-\frac{1}{2}\right)=6(1)$

$2x-3=6$

$+3 \quad +3$

$2x=9$

$x=\frac{9}{2}$

Case #2

$-\left(\frac{x}{3}-\frac{1}{2}\right)=1$

$6\left(-\frac{x}{3}+\frac{1}{2}\right)=6(1)$

$-2x+3=6$

$-3 \quad -3$

$-2x=3$

$x=-\frac{3}{2}$

b.) $\left|\frac{2x+1}{3}\right|=5$

Case #1

$\frac{2x+1}{3}=5$

$2x+1=5(3)$

$2x+1=15$

$-1 \quad -1$

$2x=14$

$x=\frac{14}{2}=7$

Case #2

$-\left(\frac{2x+1}{3}\right)=5$

$-(2x+1)=5(3)$

$-2x-1=15$

$+1 \quad +1$

$-2x=16$

$x=\frac{16}{-2}=-8$

c) $|x|=x+2$

Case #1

$x=x+2$

$-x \quad -x$

$0=2$

no solution

Case #2

$-x=x+2$

$-x \quad -x$

$-2x=2$

$x=-1$

PC11 8.2 con't... 3.

3d) $|1-2x| = x+2$

Case #1

$1-2x = x+2$

$-1 \quad -x \quad -x \quad -1$

$-3x = 1$

$x = -\frac{1}{3}$

Case #2

$-(1-2x) = x+2$

$-1+2x = x+2$

$+1 \quad -x \quad -x+1$

$x = 3$

e.) $\left|\frac{x}{3} - 1\right| = 6-2x$

Case #1

$\frac{x}{3} - 1 = 6-2x$

$3\left(\frac{x}{3} - 1\right) = 3(6-2x)$

$x - 3 = 18 - 6x$

$-x \quad -18 \quad -18 \quad -x$

$-21 = -7x$

$x = 3$

Case #2

$-\left(\frac{x}{3} - 1\right) = 6-2x$

$-3\left(\frac{x}{3} - 1\right) = 3(6-2x)$

$-x + 3 = 18 - 6x$

$+6x \quad -3 \quad -3 \quad +6x$

$5x = 15$

$x = 3$

f) $|x-2| = x$

Case #1

$x-2 = x$

$-x \quad -x$

$-2 = 0$

no solution

Case #2

$-(x-2) = x$

$-x+2 = x$

$+x \quad +x$

$2 = 2x$

$x = 1$

PC11 8.2 con't...4

4. Note $|x| = |y|$, there are 4 cases
pos x = pos y and neg x = neg y $\Rightarrow$ yield same solution
pos x = neg y and neg y = pos x $\Rightarrow$ yield same solution
$\therefore$ Only need to consider two cases.
① both sides positive
② one side positive & one side negative

a) $|2x| = |x+4|$

Case #1
$2x = x+4$
$-x \quad -x$
$x = 4$

Case #2
$-2x = x+4$
$-x \quad -x$
$-3x = 4$
$x = -\frac{4}{3}$

b) $|3x-1| = |3x|$

Case#1
$3x-1 = 3x$
$-3x \quad -3x$
$-1 = 0$
no solution.

Case#2
$-(3x-1) = 3x$
$-3x+1 = 3x$
$+3x \quad +3x$
$1 = 6x$
$x = \frac{1}{6}$

c) $|x+3| = |x-3|$

Case #1
$x+3 = x-3$
$-x \quad -x$
$3 = -3$
no solution

Case #2
$-(x+3) = x-3$
$-x-3 = x-3$
$+x+3 \quad +x+3$
$0 = 2x$
$x = 0$

PC11 8.2 con't ... 5.

4. d) $|x-5| = |5x-3|$

Case #1
$x-5 = 5x-3$
$-x+3 \quad -x+3$
$-2 = 4x$
$x = \frac{-2}{4} = \frac{-1}{2}$

Case #2
$-(x-5) = 5x-3$
$-x+5 = 5x-3$
$+x+3 \quad +x+3$
$8 = 6x$
$x = \frac{8}{6} = \frac{4}{3}$

e) $\left|\frac{x+1}{2}\right| = \left|\frac{x+2}{3}\right|$

Case #1
$\frac{x+1}{2} = \frac{x+2}{3}$
$3(x+1) = 2(x+2)$
$3x+3 = 2x+4$
$-2x-3 \quad -2x-3$
$x = 1$

Case #2
$-\left(\frac{x+1}{2}\right) = \frac{x+2}{3}$
$-3(x+1) = 2(x+2)$
$-3x-3 = 2x+4$
$-2x+3 \quad -2x+3$
$-5x = 7$
$x = \frac{-7}{5}$

f) $|2(x+1)| = -|2(x-1)|$

Case #1
$2x+2 = -(2(x-1))$
$2x+2 = -2x+2$
$+2x-2 \quad +2x-2$
$4x = 0$
$x = 0$

check:
$|2(0+1)| = -|2(0-1)|$
$|2| = -|-2|$
$2 = -2$
no solution.

Case #2
$-2(x+1) = -2(x-1)$
$-2x-2 = -2x+2$
$+2x \quad +2x$
$-2 = 2$
no solution

* Note: Absolute value will never equal a negative.

PC 11 8.2 con't...6.

5,a) $|x-10| = x^2-10x$

Case #1

$x-10 = x^2-10x$

$-x+10 \quad -x+10$

$0 = x^2-11x+10$

$0 = (x-10)(x-1)$

$x=10 \quad x=1$

Case #2

$-(x-10) = x^2-10x$

$-x+10 = x^2-10x$

$+x-10 \quad +x-10$

$0 = x^2-9x-10$

$0 = (x-10)(x+1)$

$x=10, \quad x=-1$

Check: (x=10)

$|10-10| = 10^2-10(10)$

$0 = 0$ ✓

$x=1$

$|1-10| = (1)^2-10(1)$

$|-9| = 1-10$

$9 = -9$ ×

False

(x=-1)

$|-1-10| = (-1)^2-10(-1)$

$|-11| = 1+10$

$11 = 11$ ✓

b.) $|x^2-2x| = 1$ $\qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Case #1

$x^2-2x = 1$

$x^2-2x-1 = 0$

$a=1 \quad b=-2 \quad c=-1$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-1)}}{2(1)}$

$= \frac{2 \pm \sqrt{4+4}}{2}$

$= \frac{2 \pm \sqrt{8}}{2}$

$= \frac{2 \pm \sqrt{4 \cdot 2}}{2}$

$= \frac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$

Case #2

$-(x^2-2x) = 1$

$-x^2+2x = 1$

$0 = x^2-2x+1$

$0 = (x-1)(x-1)$

$x=1$

PC11 8.2 cont... 7

5.c) $|x^2-2x+2| = 3x-4$

Case #1

$x^2-2x+2 = 3x-4$

$-3x+4 \quad -3x+4$

$x^2-5x+6=0$

$(x-2)(x-3)=0$

$x=2 \quad x=3$

Case #2

$-(x^2-2x+2)=3x-4$

$-x^2+2x-2=3x-4$

$0=x^2-2x+2+3x-4$

$0=x^2+x-2$

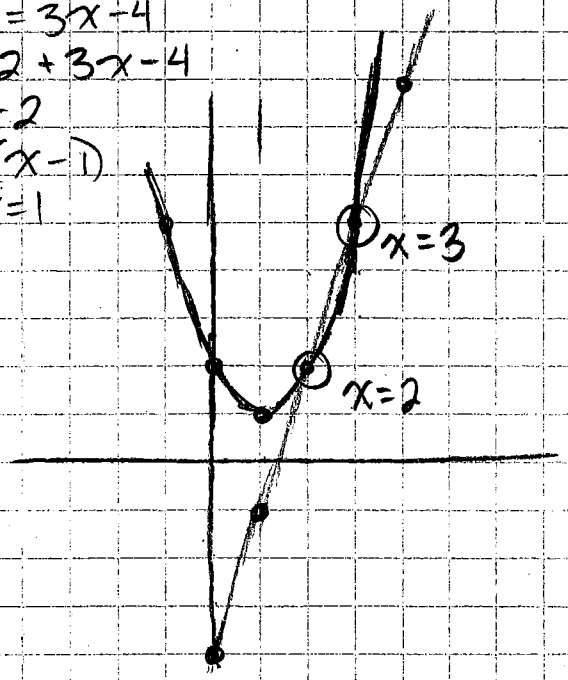
$0=(x+2)(x-1)$

$x=-2 \quad x=1$

↑ reject ↑

$\frac{1}{2}(-2)=-1$

$\hookrightarrow (-1)^2=1$



Check by graphing

$y_1 = |x^2-2x+2|$

$= |x^2-2x+1-1+2|$

$y_1 = |(x-1)^2+1|$

$y_2 = 3x-4$

d.) $|x^2-2x-16| = 8$

Case#1

$x^2-2x-16=8$

$+8 \quad -8$

$x^2-2x-24=0$

$(x-6)(x+4)=0$

$x=6, \; x=-4$

Case #2

$-(x^2-2x-16)=8$

$-x^2+2x+16=8$

$0=x^2-2x-16+8$

$0=x^2-2x-8$

$0=(x-4)(x+2)$

$x=4, \; x=-2$

Check:

$x=6$: $|(6)^2-2(6)-16| = 8$

$|8| = 8$ ✓

$x=-4$: $|(-4)^2-2(-4)-16| = 8$

$|8| = 8$ ✓

$x=4$: $|(4)^2-2(4)-16| = 8$

$|-8| = 8$ ✓

$x=-2$: $|(-2)^2-2(-2)-16| = 8$

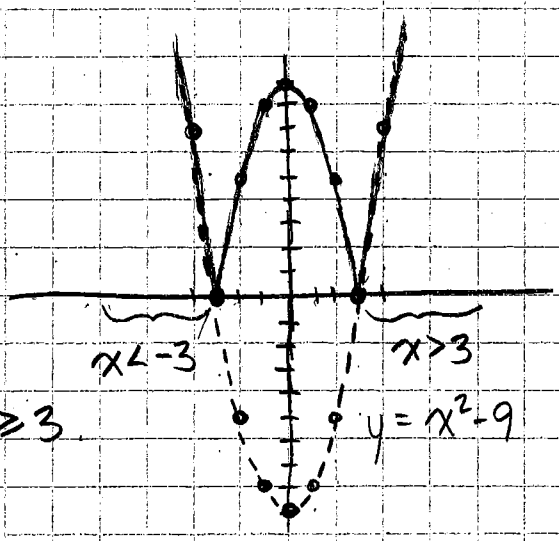
$|-8| = 8$ ✓

PC11 8.2 con't ... 8.

5. e) $|x^2-9| = x^2-9$

Graph $y_1 = |x^2-9|$

$y_2 = x^2-9$



The graphs are equal where $x \leq -3$ and $x \geq 3$.

f.) $|x-5| = x^2-8x+15$

Case #1

$x-5 = x^2-8x+15$

$-x+5 \quad -x \quad +5$

$0 = x^2-9x+20$

$0 = (x-5)(x-4)$

$x=5 \quad x=4$

Case #2

$-(x-5) = x^2-8x+15$

$-x+5 = x^2-8x+15$

$+x-5 \quad +x \quad -5$

$0 = x^2-7x+10$

$0 = (x-5)(x-2)$

$x=5 \quad x=2$

6. $|t-11.5| = 2.5$

Case #1

$t-11.5 = 2.5$

$+11.5 \quad +11.5$

$t = 14\,^\circ C$

Case #2

$-(t-11.5) = 2.5$

$-t+11.5 = 2.5$

$-11.5 \quad -11.5$

$-t = 9$

$t = -9\,^\circ C$

PC 11 8.2 cont...9.

7. Note $|x| + |y| = c$, there are 4 cases

$(+) + (+) = c$ $\quad$ $(+) + (-) = c$

$(-) + (-) = c$ $\quad$ $(-) + (+) = c$

a) $|x+1| + |x-1| = 4$

Case #1	Case #2	Case #3	Case #4
$x+1 + x-1 = 4$	$-(x+1) - (x-1) = 4$	$(x+1) - (x-1) = 4$	$-(x+1) + (x-1) = 4$
$2x = 4$	$-x-1-x+1 = 4$	$x+1-x+1 = 4$	$-x-1+x-1 = 4$
$x = 2$	$-2x = 4$	$2 = 4$	$-2 = 4$
	$x = -2$	no solution	no solution

b) $|x-2| + |x+3| = 5$

Case #1	Case #2	Case #3	Case #4
$x-2+x+3 = 5$	$-(x-2) - (x+3) = 5$	$(x-2) - (x+3) = 5$	$-(x-2) + (x+3) = 4$
$2x+1 = 5$	$-x+2-x-3 = 5$	$x-2-x-3 = 5$	$-x+2+x+3 = 4$
$-1 \quad -1$	$-2x-1 = 5$	$-5 = 5$	$5 = 4$
$2x = 4$	$+1 \quad +1$	no solution	no solution
$x = 2$	$-2x = 6$		
	$x = -3$		

c) $|x+2| + |x-3| = -2$ → Note: The sum of two absolute values (ie two positives) will never be negative.
∴ no solution

d) $|x-4| + |x+5| = 6$

Case #1	Case #2	Case #3	Case #4
$x-4+x+5 = 6$	$-(x-4) - (x+5) = 6$	$(x-4) - (x+5) = 6$	$+(x-4) + (x+5) = 6$
$1 = 6$	$-x+4-x-5 = 6$	$x-4-x-5 = 6$	$-x+4+x+5 = 6$
no solution	$-2x-1 = 6$	$-9 = 6$	$10 = 6$
	$+1 \quad +1$	no solution	no solution.
	$-2x = 7$		
	$x = -\frac{7}{2}$ → check $\left\|-\frac{7}{2}-4\right| + \left\|-\frac{7}{2}+5\right| = 6$		
	$7.5 + 1.5 = 6$		
	$9 = 6$ no solution		